SIMULATION OF AIRFLOW OVER TOPOGRAPHY WITH BUILDING STRUCTURES USING CURVILINEAR IMMERSED BOUNDARY METHOD
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Abstract
An Immersed Boundary Method (IBM) has been incorporated in a finite-difference Large-eddy Simulation (LES) code to solve the Boussinesq fluid flow over terrain using the boundary-fitted general curvilinear coordinate system. IBM has originally been developed for solving flows over complex boundary shapes on rectangular grids. In the present work it has been extended to curvilinear coordinate system in order to represent complex objects on undulating terrain, such as hills and valleys. Basic verification study has been conducted by computing laminar flows over a model terrain and rectangular objects to assess merits and efficiency of such a procedure compared with conventional methods. It is found that the present method is a good combination of two methods and is suited for calculating flow over topography with objects of various shapes.

Key words: flow over topography, IBM method, curved coordinates, LES

1. INTRODUCTION
Simulation of airflows over modern urban areas sprawling out to hilly areas like many Japanese suburbs, is often required to represent houses and building structures in addition to the undulating topography. The latter is important to reflect such general terrain effects as speed up over hills and flow reversals downhill. It is generally done by using terrain-following curvilinear coordinates (e.g. Xue et al. 2001). The former effects of the structures with angular shapes that are very important in urban areas, are hard to be represented by the curved coordinates. Recently immersed boundary (IB) methods that represent irregular boundaries in non-boundary fitting grids by means of body forces are applied in various flow simulations with irregular boundaries (e.g. Cui et al. 2000, Tseng and Ferziger 2003). This method is very effective if implemented in rectangular mesh in which the numerical algorithms are simpler than in curved coordinates and it is usually applied in rectangular grids. Flow past a circular cylinder, Fudin et al. (2000), for example, has been calculated with his method successfully in rectangular coordinates. Flow past wavy surface has also been simulated using similar method(Tseng and Ferziger 2003). One shortfall is that even with this method of enforcing boundary conditions at positions not exactly coinciding with the numerical grid, the accuracy depends on the grid spacing. Therefore the grid spacing needs to be small where high resolution is required in such areas as near the ground where flow field and the boundary geometry have small-scale variations. Furthermore, if the ground is on hills and valleys, the areas that need high grid resolution undulate depending on the terrain topography. In such a situation, the use of rectangular grid is not effective, since the ground elevation changes by large distance in the vertical direction and the wide area must be covered by fine mesh of small grid spacing if the structures on the undulating ground is to be represented adequately. The use of boundary fitting curvilinear coordinates and the immersed boundary technique can be a very effective method of representing structures on undulating terrain. It does diminish the merit of simple algorithm of immersed boundary in rectangular coordinates but the savings in the total number of grid points can be significant.

The present paper describes an attempt of implementing the immersed boundary technique in curvilinear coordinates. Similar attempt is recently proposed by Ge and Sotiropoulos (2007). We apply the direct forcing method developed by Mohd-Yusof (1997) since it does not require a free parameter and simpler than earlier feedback method. A validation is done with low-Reynolds number laminar flow and sample calculation results of flow over a real hilly suburban area with a few buildings are shown.

2. BASIC NUMERICAL METHOD
In the present work, we extend the direct forcing method developed by Mohd-Yusof (1997) to general curvilinear coordinates and also to the temperature boundary condition. It introduces extra body force to enforce the boundary condition on the boundaries that does not coincide with the grid lines. We incorporate this method in the solution method for fluid motion that follows Boussinesq approximation, so similar method is also used in the computation of the temperature field.

2.1. Equations of motion with boundary body force in curvilinear coordinates
If the sub-grid scale turbulence stress is expressed by the eddy viscosity model, the continuity equation and the equation of motion in curvilinear coordinates for Boussinesq fluid with body force can be written as
\[
\frac{1}{J} \frac{\partial (JU^k)}{\partial \xi^k} = 0
\]

and

\[
J^{-1} \frac{\partial U^i}{\partial t} + \frac{\partial (j^m U_j)}{\partial \xi^m} = - \frac{\partial}{\partial \xi^m} A^m \rho' + (v + \nu_G) \frac{\partial}{\partial \xi^m} G^{mn} \frac{\partial U_n}{\partial \xi^m} + J^{-1} \frac{\partial Z_n}{\partial \xi_j} \frac{\partial U_j}{\partial \xi_n} \left( \frac{\partial Z_n}{\partial \xi_j} \frac{\partial U_j}{\partial \xi_n} + \frac{\partial U_j}{\partial \xi_j} \frac{\partial Z_n}{\partial \xi_n} \right) + \delta_{ij} \beta (T - T_0) + f_i
\]

where \( u_i \) is the velocity components in the rectangular coordinates \( x_i \), \( U^i \) is the contravariant velocity component in the plane perpendicular to the curvilinear coordinates \( \xi^i \), \( t \) is time, \( \rho' \) is the pressure including the normal component of the sub-grid stress divided by the pressure, \( v \) and \( \nu_G \) are the molecular and eddy-viscosity coefficient, \( \beta \) is the thermal expansion coefficient, \( T \) and \( T_0 \) are the temperature at \( x_i \) and at a reference point, and

\[
J^{-1} \frac{\partial X_i}{\partial \xi_j}, \quad A^m = J^{-1} \frac{\partial Z_m}{\partial \xi_i}, \quad G^{mn} = J^{-1} \frac{\partial Z_n}{\partial \xi_i} \frac{\partial Z_m}{\partial \xi_i} + \delta_{mn} \frac{\partial Z_n}{\partial \xi_i} \frac{\partial Z_m}{\partial \xi_i}
\]

\( f_i \) is the body force applied at points near the building surfaces to enforce the boundary condition.

The equation for the temperature \( T \) with the extra heat source is

\[
J^{-1} \frac{\partial T}{\partial t} + \frac{\partial (U^m T)}{\partial \xi^m} = (\gamma + \gamma_G) \frac{\partial}{\partial \xi^m} G^{mn} \frac{\partial T}{\partial \xi^m} + J^{-1} \frac{\partial Z_n}{\partial \xi_j} \frac{\partial T}{\partial \xi_n} \left( \frac{\partial Z_n}{\partial \xi_j} \frac{\partial T}{\partial \xi_n} + \frac{\partial T}{\partial \xi_j} \frac{\partial Z_n}{\partial \xi_n} \right) + k
\]

where \( \gamma \) and \( \gamma_G \) are the molecular and eddy heat conductivity coefficients and \( k \) is used to impose the temperature boundary condition similar to the body force \( f_i \) in the momentum equation.

We solve the above system of equations with the boundary condition that is given by a wall model on the ground surface and no-slip condition on the surfaces of building structures. The former condition is applied on the ground surface given by the curved coordinates and the latter is imposed by the body force.

2.2. Numerical method

The basic numerical method we use is a fractional-step method similar to that of Zang et al. (1994) and Nakayama et al. (2001) extended to include the buoyancy and the boundary body force terms. The collocated grid arrangement is used in which the velocity components and pressure are solved at cell centers. In the first prediction step the intermediate velocity is computed by applying the Crank-Nicolson method to the diagonal terms of the viscous and the turbulent stress terms and by the Adams-Bashforth method to the convective terms. Then the pressure is calculated by solving the Poisson equation for the pressure. The temperature is computed in a single step with the Adams-Bashforth method. In both cases the boundary conditions on the ground are enforced by the direct forcing that is determined by linear interpolation.

3. VALIDATION OF THE METHOD IN FLOW PAST MODEL HILL WITH A BLOCK

First the method is applied to the flow past a two-dimensional model hill with a rectangular block placed at the top or near the bottom of the hill. Figure 1(a) shows a typical rectangular grid used in a simulation of boundary layer flows in which the horizontal gridlines are spaced closer near the ground to better resolve the near ground flow. The vertical gridlines are spaced closer near the position where the hill is located. Figure 1(b), on the other hand, is a grid based on the boundary-fitted curvilinear coordinates. In this case, the gridlines parallel to the ground is spaced closer near the ground and the hill surfaces.

A test calculation was conducted at a low Reynolds number of 100 based on the height \( H \) of the hill and the average approach-flow velocity \( U_0 \). Figures 2 and 3 show the calculation results around the hill of the case where a single block in placed on top and at the bottom of the hill in terms of the lines of constant velocity magnitude and

![Figure 1](#)
velocity vectors, compared with those obtained by the usual IBM on the rectangular grid (Figures 2(a) and 3(a)).

With the rectangular grid the region near the top of the hill is resolved poorer but the results obtained with the terrain-fitted curvilinear coordinates resolve both area well and the effects of the rectangular block is represented consistently well.

4. APPLICATION IN PREDICTION OF WIND FIELD OVER REAL TOPOGRAPHY WITH BUILDINGS

The same calculation method is now applied to compute the airflow over real topography with several rectangular buildings on the hillside. Figure 4 shows the numerical grid constructed from the geographic data of a suburb of Toyama where wind field was to be simulated. The distribution of the local wind speed and the direction is to be obtained in this hilly area with a few distinguished building structures. The area of simulation is about 5 km square and the elevation difference between the valleys and the hills is about a few hundred meters. The computational grid is 120×120×30 resolving to about a few meters near the ground but the grid spacing is few hundred meters near the top of the simulation region. A power-law velocity profile is assumed at the inflow section and iso-thermal condition is assumed throughout the calculation region for this calculation. Figure 4(a) shows streamlines near the ground (about 10 m from the surface) in the region indicated by the rectangle in Figure 4(b) obtained by the calculation using the terrain following coordinates ignoring the building structures. Figure 4(c) shows similar calculation results obtained by using the same grid but with the buildings represented by the IBM method. It is seen that the effects of the structures on the curving streamlines are represented reasonably well. In both cases the usual log-law is applied to the first calculation point on the curved terrain and no-slip condition is applied on the surfaces of the buildings in the second case.

5. CONCLUSION

An Immersed Boundary Method has been incorporated in a finite difference LES code in order to simulate the flow over hills with building structures. It is found that the no-slip boundary condition for bluff obstacles like buildings can be easily incorporated with IBM while the ground boundary condition that requires an imposition of the wall and roughness model can better be prescribed on a boundary-fitted curved coordinates. Furthermore the terrain-following coordinates allow higher grid density in the layer near the curved ground where higher resolution is desired. Overall, the present method is found to be a good combination of the two numerical techniques and can be an effective method for simulation of flow over mountainous terrain with distinguished angular structures that are seen in many suburbs of developing cities in Japan and probably in many other countries.
Figure 4. Example calculation of wind field in a suburb of Toyama. (a) terrain-fitting computational grid, (b) the calculated streamlines showing the flow up the valley to the hill top ignoring the buildings, (b) similar calculation with the building represented by the IBM.

References