PREDICTIONS AND EVALUATIONS OF VARIOUS TURBULENT BOUNDARY LAYERS USING LES

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Abstract

This paper presents evaluations for the prediction precision of various SGS models of LES under the simple condition of urban environment, i.e., various thermally-stratified turbulent boundary layers are calculated by evaluated LES models, namely, the Smagorinsky and the Mixed-Time-Scale models, the representative SGS model for the velocity field, and the constant turbulent Prandtl number model for the thermal field. These models are evaluated in comparison with a DNS database, in which the problems of LES in the prediction of various thermally-stratified boundary layers are elucidated. On the other hand, in order to evaluate the prediction precision of various models in a complicated turbulent flow field, the LES and the hybrid LES/RANS models are conducted in a turbulent boundary layer over a 2-dimensional hill. These results are assessed using newly-conducted DNS results of 2-dimensional hill, and prediction precision of these models in the calculation of complicated turbulent flow field are clarified.

Key words: LES, Hybrid LES/RANS, Boundary Layer, 2-dimensional hill, Evaluation

1. INTRODUCTION

LES (Large eddy simulation) is the effective calculation technique for a numerical analysis of a turbulent heat and mass transfer in an urban environment, because it uses smaller grid numbers than those of DNS (direct numerical simulation). The LES model has been mainly improved and proposed in the engineering field, and the SGS model is especially useful to calculate actual engineering problems (e.g., Inagaki et al., 2005). On the other hand, the spatially-developing turbulent boundary layer is considered the principal flow in an urban environment, but recent LES models have almost never been evaluated using the DNS of a boundary layer. In order to adequately predict turbulent heat and mass transfer in an urban environment including an effect of complex terrain, an LES model should appropriately predict a near-wall phenomenon. Thus, LES models must be evaluated using near-wall detailed turbulent data. DNSs of boundary layers have been conducted to determine the near-wall data (Hattori et al., 2007), in which understanding of the turbulence phenomena in the thermally stratified boundary layer has been improved. Therefore, the first objective in this study is to evaluate the prediction precision of various SGS models of LES under the simple condition of urban environment, in which various thermally-stratified turbulent boundary layers are calculated by the evaluated LES models in the velocity field as follows: Smagorinsky model (hereinafter referred to as S model) (Smagorinsky, 1963), which is the representative SGS model, and the Mixed-Time-Scale model (hereinafter referred to as MTS model) (Inagaki et al., 2005), which was developed for engineering applications. For the thermal field, the constant turbulent Prandtl number model is evaluated. These models are evaluated in comparison with a DNS database, in which problems of LES in the prediction of various thermally-stratified boundary layers are elucidated.

Next, in order to assess the prediction of a complex turbulent boundary layer using LES, the LES is adopted to predict a turbulent boundary layer over a 2-dimensional hill which may affect the urban environment. On the other hand, for a prediction of turbulent flow with separation and reattachment, it may be difficult to predict such flow due to the near-wall treatment of LES. Thus, a hybrid LES/RANS (HLR) technique has been developed for proper prediction of a turbulent flow with separation and reattachment. Thus, the HLR technique is also applied to appropriately predict the turbulent boundary layer over a 2-dimensional hill. In order to evaluate these models in the field, DNS is newly carried out to obtain detailed turbulent quantities of the turbulent boundary layer over the 2-dimensional hill. Results of the LES and HLR are then evaluated using DNS data.

2. NUMERICAL PROCEDURE

The governing equations used in DNS are the Navier-Stokes equation without buoyancy, the continuity equation for the velocity field, and the energy equation for the thermal field, in which incompressibility is assumed as follows:

\[ \bar{u}_{ij} = 0 \]  
\[ \partial \bar{u}_{i}/\partial t + \bar{u}_{j}(\bar{u}_{i,j}) = -\bar{p}_{i} + (1/Re_{\delta_{i}})\bar{u}_{i,j} - \tau_{ij,j} + \delta_{ij}Ri_{\delta_{i}} \bar{\theta} \]  
\[ \bar{\theta}_{i} + \bar{u}_{i}\bar{\theta}_{j} = [1/(PrRe_{\delta_{i}})]\bar{\theta}_{ij,j} - q_{ij} \]  

where the Einstein summation convention applies to repeated indices, and a comma followed by an index indicates differentiation with respect to the indexed spatial coordinate. \( \bar{u}_{i} \) is the dimensionless velocity component in \( x_{i} \) direction, \( \bar{p} \) is the dimensionless pressure, \( \tau \) is the dimensionless time, and \( x_{i} \) is the dimensionless spatial coordinate in...
the \(i\) direction, respectively. All equations are non-dimensionalized by the free stream velocity, \(\bar{U}_0\), the momentum thickness, \(\delta_{2,m}\), and the temperature difference between the wall and the free stream, \(\Delta\Theta\), at the inlet of the driver part.

In case of DNS, turbulent stress, \(\tau_{ij}\), and turbulent heat flux, \(q_{ij}\) in Eqs. (1) and (2) are equal to zero, and \(\bar{\tau}\) means an instantaneous value in equations. The filtered turbulent quantities are used in equations for the LES (Inagaki et al., 2005), and \(\bar{\tau}\) means a GS component. Thus, \(\tau_{ij} = \bar{\pi}_{ij} - \bar{u}_i\bar{u}_j\) is an SGS stress tensor, and \(q_{ij} = \bar{u}_i\bar{\theta}_j - \bar{\theta}_j\bar{u}_i\) is an SGS heat flux, which represent the effect of SGS components, and should be modeled. In case of RANS in HLR, the Reynolds-averaged turbulent quantities are employed in equations (Wu, Hattori and Nagano, 2006), and \(\bar{\tau}\) means an ensemble-average or a time-average value. Therefore, \(\tau_{ij} = \bar{\pi}_{ij}\) is Reynolds stress, and \(q_{ij} = \bar{u}_i\bar{\theta}_j\) is turbulent heat flux. In the present study, the gradient diffusion type model is used for the modeling of \(\tau_{ij} = -2\nu\bar{S}_{ij} + 2(3/\delta)\bar{k}\) and \(q_{ij} - \kappa(\bar{\theta}_j\bar{u}_i)\), where \(\nu\) and \(\kappa\) are the eddy diffusivity for momentum and heat, respectively, \(k\) is a turbulence energy or a turbulence energy of SGS component, and \(\bar{\pi}_{ij} = (\bar{u}_i\bar{u}_j + \bar{\theta}_i\bar{w}_j)/2\). In order to solve \(\nu\), the MTS model (Inagaki et al., 2005) is adopted for the velocity field of LES as \(\nu = C_{MTS}\alpha_t\), the low-Reynolds-number one-equation model (Wu, Hattori and Nagano, 2006) is employed for the RANS part of HLR, and the MTS model is also used for the LES part of HLR. As for the eddy diffusivity for heat, \(\kappa\), of both LES and RANS, the zero-equation model is adopted as \(\kappa = \nu_t/P_{\kappa}\), where \(P_{\kappa} = 0.9\). For the switching indicator between LES and RANS of HLR, the function (Nikitin et al., 2000), \(\delta \equiv \min(d, C_{DES}\Delta), \Delta \equiv \max(\Delta_x, \Delta_y, \Delta_z), \) is applied, where \(d\) is distance from the nearest wall, \(\Delta_x, \Delta_y\) and \(\Delta_z\) are a grid width of \(x, y\) and \(z\)-direction, respectively, and model constant is \(C_{DES} = 0.65\). In the calculation of HLR, RANS is conducted when \(\delta = d\), and LES is conducted when \(\delta = C_{DES}\Delta\).

Figure 1 shows a schematic of the thermally-stratified boundary layer and the boundary layer over 2-dimensional hill and the coordinate system. The present calculations based on the high-accuracy finite-difference method (Hattori et al., 2007; Hattori and Nagano, 2003) are carried out. As for the thermally-stratified boundary layer using LES, Reynolds number based on the free stream velocity and the momentum thickness at the inlet of the driver part, \(Re_{x,w}=1000\), Prandtl number, \(Pr = 0.71\), and the Richardson numbers, \(\text{Bi}_{x,w} = k\beta\delta_{2,m}\Delta\Theta U_0^2/\nu\), are set to 0.06 (SSBL) and 0.01 (WSBL) for the stable stratification boundary layer cases, 0 for the neutral boundary layer (NBL) case and -0.01 for the unstable stratification boundary layer (UBL) case. The grid numbers are \(96 \times 64 \times 64\) for the main-simulation part of NBL, UBL and WSBL cases, \(192 \times 64 \times 64\) for the main-simulation part of SSBL case. Note that the grid number of DNS is \(384 \times 128 \times 128\) (Hattori et al., 2007).

As for the 2-dimensional hill, the hill shape is determined by the function as \(h(x) = h_0\cos^2(\pi x/2L)(-1 \leq x/L \leq 1)\), where \(h_0 = 3\delta_{2,m}\) and \(L = 15\delta_{2,m} = 5h\). The DNS of boundary layer over 2-dimensional hill under conditions of Reynolds number, \(Re_{x,w}=300\), Prandtl number, \(Pr = 0.71\), and the Reynolds numbers based on the hill height becomes \(Re_{\bar{h}} = 900\). In order to obtain the shape of a 2-dimensional hill, the immersed boundary condition (Fadlun et al., 2000) is used for DNS, and the boundary-fitted condition in the generalized coordinate system is used for LES and HLR (Hattori and Nagano, 2003). The grid numbers are \(192 \times 128 \times 64\) for the driver part of DNS, and \(344 \times 128 \times 64\) for the main part of DNS, with \(86 \times 32 \times 64\) for the driver part of LES and HLR, and \(172 \times 32 \times 64\) for the main part of LES and HLR. The boundary conditions for the velocity field are the non-slip conditions on the walls, and \(\delta u/\partial x = 0, \delta w/\partial y = 0, \partial v/\partial y = -(\partial u/\partial x + \partial w/\partial z)\) on the upper boundary (free stream). At the outlet of both parts, the convective boundary conditions are applied, and periodic boundary conditions are used in the spanwise direction.

![Schematic of computational fields and coordinate system](image-url)
3. RESULTS AND DISCUSSION

First, LES evaluation results in a thermally-stratified boundary layer are shown in Fig. 2. The case of NBL is indicated in Figs. 2(a) and (b). For the velocity field, since Reynolds shear stresses are slightly overpredicted by both models, the mean velocities are also overpredicted near the wall. In particular, S model gives overprediction of Reynolds shear stress over the boundary layer thickness. It can be seen that mean temperature and wall-normal turbulent heat flux are overpredicted near the wall. Figure 2(c) shows the mean velocity and Reynolds shear stress in the case of UBL. In the unstable stratification boundary layer, turbulence tends to be enhanced thereby, and Reynolds shear stress becomes large in comparison with the case of NBL. The LES models can capture the tendency of this phenomenon, but predictions of both models are not in agreement with DNS data near the wall. In the weak stable stratification boundary layer (WSBL), the turbulence tendency indicates a converse phenomenon. Also, LES models give a similar tendency, but it can be seen that the Reynolds shear stresses are overpredicted in comparison with DNS data as shown in Fig. 2(d). The case of a strong stable stratification boundary layer (SSBL) is shown in Fig. 2(e) and (f), where the MTS model is only included in figures. In this case, it is found that the counter gradient diffusion phenomenon occurs in both the velocity and thermal fields (Hattori et al., 2007). The MTS model can capture the counter gradient diffusion phenomenon in both fields, and the predictions almost agree with DNS data.

Next, the results of calculations of a boundary layer over the 2-dimensional hill are shown in Figs. 3 and 4. In order to observe phenomena of a boundary layer over the 2-dimensional hill, the DNS result for a velocity field is demonstrated in Fig. 3, in which the mean velocity and Reynolds shear stress are indicated. It can be observed that the near-wall Reynolds shear stress is suppressed due to the effect of the curvature in front of the hill. Downstream of the crest of the hill, the flow has separation and reattachment in back of the hill, where a recirculation region clearly exists. The Reynolds shear stress pronouncedly increases above the recirculation region due to the occurrence of a large velocity gradient. The maximum points of Reynolds shear stress, however, do not agree with the zero velocity gradient.

Fig. 3: Distributions of turbulent quantities in 2D hill boundary layer
Using the present DNS, the performances of LES and HLR are verified. The reattachment points estimated in calculations are shown in Table 1, in which the predicted reattachment point of HLR is located near the DNS result. Since the HLR uses the same LES model in the region away from the wall, this improvement in prediction is obviously achieved by the RANS model which is adopted near the wall. Although the turbulent quantities shown in Fig. 4 are approximately predicted by LES, a slight disagreement with the DNS results in the vicinity of the wall of LES results in an overprediction of the reattachment point. A large disagreement between the LES and HLR predictions, however, can be found near the base of the hill due to an irregular prediction of Reynolds shear stress. Since these disagreements exist in the recirculation region, the turbulence model should be adequately improved.

4. CONCLUSIONS

Calculations of various turbulent boundary layer flows using LES are carried out to obtain knowledge of accurate LES performance, in which the LES results are evaluated using a DNS database. In the cases of thermally-stratified boundary layers, two SGS models are assessed, and appropriate results of LES for the calculation of such boundary layers are obtained. Also, DNS, LES and Hybrid LES/RANS of turbulent boundary layers with heat transfer over a 2-dimensional hill with a curvature wall shape, and detachment and an reattachment phenomena, are carried out. The present DNS indicates the fundamental and detailed characteristics of turbulent boundary layers with heat transfer over a 2-dimensional hill. LES and Hybrid LES/RANS are conducted to evaluate their prediction accuracy in such complex flow. The result of evaluations indicates that the HLR method is effective for near-wall prediction, in which HLR has a more accurate prediction performance of the recirculation region than LES.

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References


